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SUPPLEMENT 1

SIMULTANEOUS APPLICATION OF STATIC AND DYNAMIC LOADS ON SONIC FATIGUE TEST ARTICLES

SUPPLEMENT 1. CONSIDERATIONS OF THE THEORETICAL ASPECTS OF ACOUSTIC FATIGUE

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SUPPLEMENT 1

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FOREWORD

This report was prepared by Northrop Norair, a Division of Northrop Corporation, Hawthorne, California, under Contract AF 33(657)-8759. The project was initiated by the Air Force Flight Dynamics Laboratory, Vehicle Dynamics Division, Aero-Acoustics Branch under Project Number 4437, "High Intensity Sound Environment Simulation," Task Number 443703, "Investigation of Methods for Simultaneous Application of Static and Dynamic Loads on Sonic Fatigue Test Articles," with D. N. Wolf as project engineer. This report covers work performed during the period from June 1962 to June 1963 by the author, S. R. Valluri, Firestone Flight Sciences Laboratory, California Institute of Technology, Consultant to Northrop Norair. It has been assigned Report No. NOR 63-196 for internal control.

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ABSTRACT

A theory of acoustic fatigue is proposed which is based on a general theory of fatigue of metals. The theory is based on the proposition that the problem is better understood if attention is focused on the growth of the crack which is responsible for the ultimate failure. One significant manner in which the present theory differs fromothers is that it invokes a stress dependent limiting condition on the crack growth to determine the cycles for failure. Crack growth expressions are derived on a semi-intuitive basis and these are used for defining equivalent damage at different stress ~~levels~~. Representing the result of acoustic input as one of random stress response, expressions are derived for the prediction of time to failure. Indications are that the rate at which the damage is accruing in acoustic fatigue is strongly dependent upon the low stress end of the distribution function; and the failure conditions and the associated probabilities of failure are strongly determined by the high stress end; that is, the tail of the distribution function. Another important result of the analysis is that in any acoustic fatigue simulation tests, one has to take into account the design mean stress for which the structure is designed. This design mean stress not only influences the total time to failure but it also strongly affects the probabilities of failure by essentially changing the origin of the distribution function curve.

This technical documentary report has been reviewed and is approved.

HOWARD A. MAGRATH
Chief, Vehicle Dynamics Division

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LIST OF SYMBOLS

a, a_T	Modified Griffith exponent for static strength of ductile materials
b	Burgers vector
C	Empirical constant in the expression for crack propagation in fatigue
g	Exponent in the relation for mean stress and endurance limit in fatigue
K_n	Natural (plastic) stress concentration factor for the stress in the plastic enclave
K_{Tn}	Theoretical elastic stress concentration at the crack tip
K_1, K_2	Empirical constants
k	Boltzmann constant
2ℓ	Dominant crack length at any instant in acoustic fatigue
$2\ell_o$	Initial crack length of a dominant crack
L_m	Mean grain size
n	Number of cycles of fatigue stress
n_r	Number of cycles at a reference stress σ_r to cause catastrophic propagation for an applied stress $\sigma_{q\ell}$
Δn_q	Number of cycles applied at stress σ_q and frequency ω_q
(Δn_{rq})	Equivalent number of cycles at stress σ_r to cause the same damage as Δn_q cycles at stress ω_q
Δn_{erq}	Equivalent number of cycles at the reference stress to cause the same damage as ω_q cycles applied at the r.m.s. stress $\bar{\sigma}_q$; the associated distribution function at the r.m.s. stress $\bar{\sigma}_q$ is the Rayleigh distribution function (see equation 22)
N_o	Nucleation period in low stress level fatigue
P	Rayleigh distribution function, defined by equation 20

$P(f)$	Probability of failure
q	A suffix associated with the acoustic stress response
R_1	Test stress ratio
T	Process temperature in degrees absolute
t_A	Expected time to failure
t	Time
t_r	Time to failure at the reference frequency ω_r and the reference stress σ_r when the instantaneous applied stress is σ_q; t_r may also be looked upon as the time when the instantaneous residual static strength is $\sigma_{q\ell}$
t_{er}	Defined by equation 24
t_{erq}	Equivalent time during which the same amount of damage is caused as the application of stress $\overline{\sigma}_q$ with the associated distribution function for unit standard time
t_o	Integration constant
W	Panel width
α, γ	Constants for the Ramberg-Osgood stress strain relation
β	Constant
ϵ_p	Strain corresponding to the stress σ_p
λ	$= \frac{2\ell}{W}$
σ	Positive stress amplitude in the basic equation for crack propagation in fatigue
σ'	Negative stress amplitude
σ_{cr}	Critical stress for buckling of a cylindrical shell of thickness to radius (t/R)
σ_{io}	Proportional limit
σ_i	Endurance limit in acoustic fatigue, defined arbitrarily for non-failure for an arbitrarily large period of time

σ_m	Mean stress in acoustic fatigue
σ_n	Net section stress in the panel in the plane of the crack
σ_u	Ultimate tensile strength of the material under consideration
σ_p	$= \sigma_n K_n$
σ_q	Instantaneous positive stress amplitude at frequency ω_q
σ_r	Reference stress arbitrarily chosen
$\bar{\sigma}_q$	The r.m.s. stress at the frequency ω_q
$\sigma_{q\ell}$	An arbitrarily large stress used in the discussion of the probability of failure at any instant
σ_{mi}	Endurance limit corresponding to a mean stress σ_m
σ_{mq}	Mean stress at the frequency ω_q at which an r.m.s. stress peak $\bar{\sigma}_q$ occurs
σ_{mr}	Mean stress at the frequency ω_r, chosen as a reference frequency; it is assumed $\sigma_{mr} = \sigma_{mq}$
σ_{mqi}	Endurance limit at the frequency ω_q corresponding to a mean stress σ_{mq}
σ_{mri}	Endurance limit at the frequency ω_r corresponding to a mean stress σ_{mr}
μ	Poisson's Ratio
$\phi(\sigma_q)$	Cumulative probability for the occurrence of stress σ_q or smaller for the r.m.s. stress $\bar{\sigma}_q$ at the frequency ω_q
ω	An arbitrary frequency
ω_q	A frequency at which an r.m.s. stress peak $\bar{\sigma}_q$ occurs
ω_r	A reference frequency

I INTRODUCTION

During the last decade, with the advent of jets and rockets, the problem of acoustic fatigue has assumed considerable importance in structural design. Fatigue failures due to acoustically induced vibrations have been sufficiently common to warrant serious consideration during the initial stages of design. The problem of acoustic fatigue has three essential features. The first is a description of the acoustic field. The second is description of the vibratory stress response of the structure to this field. The third is a description of the damage resulting from the response of the structure to this vibratory stress. The result of this response may, generally speaking, be divided into two general kinds. In the first instance, the vibratory field may be expected to produce a submicroscopic crack field which may conceivably reduce the stiffness of the structure after a certain period of time much like creep induced buckling. The second is the process wherein one of the cracks becomes dominant in the sense that its length and rate of crack propagation exceed that of others. As the dominant crack continues to propagate, the residual cross sectional area continues to decrease. This results in a decrease of the static strength of the subject structure. When the applied stress just exceeds this residual strength, catastrophic failure may be expected to occur. It will be noted that the decrease in the residual cross section due to the propagation of the dominant crack also results in a reduction of stiffness. This again raises the possibility of time dependent decrease in stiffness and the possibility of buckling under a compressive field. Fortunately, the propagation of the crack is accented by the presence of tensile stress fields and inhibited by the presence of compressive stress fields. While we will present later some model data to show the decrease in stiffness leading to buckling under a compressive load, it appears that we may not have to worry seriously about it, since the crack length needed to cause significant decrease is fairly large.

The primary purpose of this report is a basic study of the damage problem under acoustically induced vibrations resulting in a continuous decrease in strength. We shall discuss the acoustic fatigue problem as fundamentally similar to the general problem of fatigue damage in metals and treat it as such. This would require developing a set of equations to describe the problem of fatigue of metals adequately. Within the framework and perspective offered by this set of equations, we shall make appropriate assumptions and thereby obtain equations by means of which we can treat the acoustic fatigue problem. The advantage of such an approach is that we will be continuously aware of the limitations on the applicability of the results thus obtained. Thus, if at a later stage we wish to refine the calculations or change one or more of the equations to better describe the general problem of fatigue, we will be in a position to do so. It is suggested that such an approach is desirable at present, since the acoustic fatigue problem is not too well defined in many respects and there is no point in trying to be more accurate than the load inputs warrant. But as the first two aspects are better understood we may think in terms of making refinements in our models and the resulting calculations.

One of the fundamental features of the acoustic stress input is its essentially random nature. This will be obvious if we observe the strain gage output in a panel subjected to an intense acoustic field. Because of the extreme complexity, if not of

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the impossibility of a suitable mathematical description of such data, the current practice is to describe the output in terms of the rms stress versus frequency and in turn, a description of the rms stress in terms of its frequency distribution curve. Let it first be understood that while such a description may be adequate for treatment of some aspects of acoustically induced vibrations, there are sufficient reasons to believe, from the standpoint of the physical nature of the fatigue damage, that it is insufficient to describe the acoustically induced fatigue damage. Except for one paper of Weibull in a recent symposium, on acoustic fatigue[1] and some comments by Schjelderup in the same symposium, not enough attention has been given to this problem. Both the range of stress and stress amplitude play a significant part in fatigue damage. The range of stress and the number of associated cycles determine the progress of damage in terms of crack growth and the peak stress amplitude determines the instant when the crack becomes catastrophic. In acoustic fatigue where these two things are continuously changing any mathematical description of the stress inputs that does not take this aspect into consideration is liable to be seriously in error. Since the purpose of the present report is an analysis of the physical damage, we shall, as a matter of practical necessity, follow the current practice of description. If subsequent investigations point out a better method of description of the load inputs, it will be a simple matter to incorporate it into the method of analysis proposed here.

Considering the essentially random nature of the acoustic input, it is surprising that, almost uniformly, the present theories of acoustic fatigue propose an equation for rms stress versus cycles to failure as a solution to the problem. It does not seem to be particularly meaningful to start with inputs that are probabilistic in nature and present solutions which do not reflect this aspect. The reason for this contradiction of terms of course is the currently followed methods of treatment of cumulative damage in fatigue, where the end conditions such as cycles to failure are used as fundamental parameters. A little reflection will show that, especially where the loads are continuously varying, such a treatment is unrealistic. Whether failure occurs at any stage or not, is dependent upon whether or not the applied load at any instant exceeds the remaining strength of the structure. As the acoustic input persists, the crack continues to grow in a random manner; the residual strength continues to decrease in a random manner. The applied load is random to start with. Therefore it is obvious that the probability of failure continues to increase and is strongly dependent upon the cumulative probability of the load input. In the treatment of the problem we propose here, we shall try to take into account this inherently random nature of the acoustic input.

Several publications have appeared during the last few years in the field of acoustic fatigue. An excellent summary of the state of the art appeared in the symposium[1] mentioned earlier. The description of the acoustic field is properly a field of aerodynamics. The stress response of the structure to the acoustic field is essentially an aspect of structural response to random excitation. It is the third we shall be concerned with here; namely, given the stress input we shall try to predict the time to failure and the associated probability of failure for a given rms stress versus frequency curve.

A theory of high stress level fatigue was proposed by the writer[2,3], and in a paper presented[4] by Valluri et al, some basic modifications have been proposed to make the theory more consistent and make it applicable in a formal manner to the low stress level regime also. The theory is based on the proposition that it is more meaningful to discuss the fatigue problem from the standpoint of crack growth and that the stage at which the specimen fails is determined by a stress dependent limiting condition on the crack growth. It is considered not particularly meaningful, to take the cycles to failure which is essentially an end condition of the process as

a fundamental parameter. It is proposed instead, that the residual strength of the specimen as the crack continues to grow, is a more fundamental and meaningful parameter. The theory is oriented towards the estimation of strength at any stage during fatigue and not the cycles at the instant of failure. Some of the assumptions made in the development of the equations to describe the process may very well be questioned and argued at length. But it is suggested that the residual strength oriented treatment of the fatigue problem is fundamentally sound and physically more satisfying than cycles to failure oriented treatment. And since there is considerable evidence available that the remaining (residual) strength at any crack length for a given crack geometry is uniquely determined, the strength oriented discussion of the fatigue problem may be reduced to one of crack length based discussion. This has considerable advantage, as measurement of a crack length during fatigue is a non-destructive process and is easily amenable to testing techniques, whereas determining the residual strength by tests destroys the specimen.

It is assumed for the purposes of the present discussion that the following information is available. 1) That the general localities where the acoustic fatigue cracks occur are known. 2) The dynamic response of the strain gages mounted at these points for test purposes is known. 3) Furthermore, since the dynamic response, as it is, is much too complicated for analytical treatment, in line with current (but uncertain!) practice, it is assumed that the response of the gages is passed through a narrow band wave analyzer and the output recorded on an rms voltmeter and thus obtain a rms stress versus frequency curve. It is evident that a distribution of stress pulses is associated with the rms stress peaks. A knowledge of them may be obtained by sending the signal from the strain gage through a narrow band analyzer and a suitable gate circuit and electronic counter; the gate circuit being arranged so that it accepts signals that fall in unit time in the stress amplitude range σ and $\sigma + d\sigma$. Present indications are, that to a fair degree of approximation the associated distribution function of the stress amplitudes is the Rayleigh distribution function. 4) We shall therefore assume that, as far as the fatigue damage problem is concerned, the rms stress curve can be represented by a mean stress and sharp rms stress peaks as shown in Figure 1 and that the Rayleigh distribution function is appropriate. One of the important consequences of the present theory is that while the crack growth is determined more by the low stress end of the distribution curve, the catastrophic failure itself is strongly determined by the high stress end. Therefore, at best, the acoustic fatigue failure problem can be solved with no more accuracy than the accuracy with which the tail of the distribution is determined. This is important, since there are indications that the Rayleigh distribution function does not represent the tail of the observed distribution to a reasonable degree of approximation.

It may also be noted in passing that, howsoever complex the acoustic field is, the structure itself (which is typically a reinforced panel) acts as a strong filter with the result that one in general observes only a few peaks closely related to the natural modes of vibration of the structure. The significant rms stress peaks rarely exceed beyond one decade of frequency range. And since it can be shown[5,6] that the change in fatigue life due to a decade change in frequency is negligible at room temperature, we shall neglect the frequency effect in the acoustic fatigue problem.

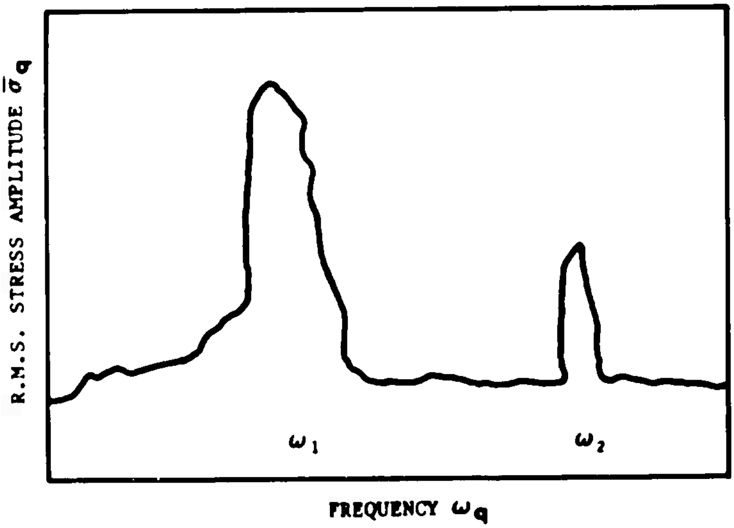

FIGURE 1A SCHEMATIC REPRESENTATION OF TYPICAL R.M.S. STRESS RESPONSE IN ACOUSTIC FATIGUE

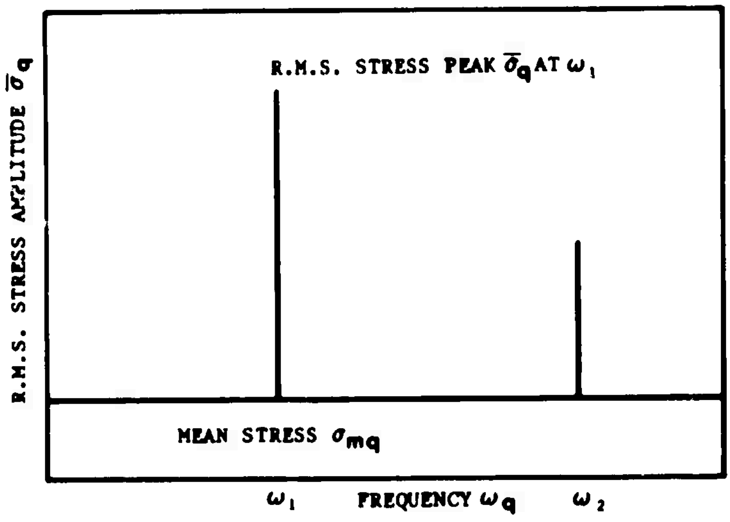

FIGURE 1B IDEALIZED STRESS RESPONSE CURVE USED IN THE DISCUSSION OF ACOUSTIC FATIGUE

II GENERAL IDEAS CONCERNING A THEORY OF FATIGUE

Any theory of fatigue which is physical and sufficiently basic must stem from the dislocation theory of metals which discusses behavior of metals in terms of lattice imperfections. Of necessity, the engineering theories of fatigue are macroscopic theories and any attempt to bridge the gulf between the two levels of approach will be to some extent arbitrary and intuitive in nature. Arbitrary as it may be, the advantage of a theory that has a reasonable physical model is that one is in a position to account for the damage, as the parameters change, in a satisfactory manner.

The writer presented earlier a theory of high stress level fatigue[2,3] based on the proposition that there exists a dominant crack whose growth controls the fatigue damage and that an independent, stress dependent, limiting condition is necessary on the crack growth to define failure. Furthermore, it was postulated that the growth of the dominant crack is facilitated by the formation and growth of submicroscopic cracks in the plastic enclave. The formation of these cracks in turn was argued to be dependent upon the pile up of dislocations due to Frank-Read sources and the extent of to and fro movements of dislocations which is dependent upon the stress range. On such a basis the following expression is formulated

$$\frac{d\ell}{dN} = C\,\frac{L_m}{b}\,\ell\,\frac{\left(\dfrac{\sigma-\sigma_i}{E}\right)^2\left(\dfrac{\sigma-\sigma'}{\sigma_i}\right)^2}{\log\left(\dfrac{\sigma-\sigma_i}{K}\right)} \tag{1}$$

where ℓ is the crack length, L_m is the mean grain size, b is the Burgers vector, σ is the applied stress, $(\sigma-\sigma')$ is the stress range, E is the Young's modulus, σ_i is an internal stress presumed to oppose the motion of dislocations and identified in the theory with the endurance limit and C and K are empirical constants. The logarithmic term in the denominator was obtained on the basis that the crack growth is not continuous and that there are active and dormant periods associated with the crack growth. The logarithmic term obtained on a rate process assumption was assumed to represent the period of dormancy during which the enclave was presumed to get strain hardened; this hardening accompanied by the formation of submicroscopic cracks. In a discussion following the presentation of the paper of reference 3 Forsyth pointed out that, more often than not, the crack growth is continuous, there being a one to one identification with the load pulses and striations. Since then several other investigators made the same observations. Correspondingly in the equation 1 above $\Delta N \approx 1$ instead of $\log\left(\frac{\sigma-\sigma_i}{K}\right)$ to give

$$\frac{d\ell}{dN} \approx C\,\frac{L_m}{b}\,\ell\left(\frac{\sigma-\sigma_i}{E}\right)^2\left(\frac{\sigma-\sigma'}{\sigma_i}\right)^2 \tag{2}$$

In other words, it is now implied that the hardening process is still continuous but that it starts as soon as an element of material enters into the plastic enclave. By the time it approaches the crack tip, the hardening process will have been completed and the crack will extend through an incremental distance equal to the width of the striations.

In such a scheme, it must be noted that equation 2 does not say anything about how the crack started. It is concerned only with its growth. In high stress level fatigue, a crack with a well defined plastic enclave is presumed to be developed in the very first cycle itself and hence the whole fatigue life is concerned with crack growth. However, in low stress level fatigue, there is no well defined crack until some time, this initial (nucleation) period being larger, the smaller the stress level. In low stress level fatigue, what Wood[7] calls fissurization may still be taking place. But it is suggested that here too the failure may still be attributed to the growth of a dominant crack whose growth may be somewhat more easy due to the fissurization.

In a recent paper, Valluri et al[4] have proposed some further modifications of equation 1 in addition to that implied in (2), in order to make it more compatible with observations. It was originally suggested that σ (and σ') are the stresses at infinity. A little reflection will show, however, that the submicroscopic cracks in the enclave will see, not the stress at infinity, but the local stress. Accordingly, σ is strictly speaking σ_p, the local stress in the plastic enclave which really determines whether submicroscopic cracks form or not. This interpretation is also consistent with the meaning assigned to the term σ_i. σ_i is the stress that must be overcome before the dislocations can move. The stress they "see" is the local stress in their vicinity and not the stress at infinity. If the stress in the plane of the crack, far away from the crack is σ_n and the natural stress concentration factor is K_n, then

$$\sigma_p = K_n \sigma_n \tag{3}$$

A second difficulty with equation 2 is that concerning σ_i. The internal stress in the dislocation theory is generally identified with the proportional limit; that is, the stress above which a number of dislocations move from their sources and cause "observable" plastic deformation. If this identification is retained in fatigue, one faces the contradiction of not having any fatigue failures for all stresses below σ_i. To get around this contradiction, Valluri invoked Bairstow's hypothesis that the natural proportional limit and the endurance limit are the same and hence identify σ_i with the endurance limit and determine it from fatigue tests. This is patently an unsatisfactory solution, for it is a well known fact that if one continues a test long enough, except for interstitial solid solutions, fatigue failures will always tend to occur. This is partly due to the fact that σ_i has a statistical spread associated with it. Probably, a more important fact is that the state of the material in a specimen subjected to fatigue does not remain static. Howsoever small, dislocation movements leading to rearrangement, which generally leads to a decrease in the mean internal stresses, seems to be fairly common. In low stress level fatigue, fissurization preceded by fine slip seems to be a characteristic feature for a substantial period of fatigue life. Regions of fine slip which gradually intensify to give rise to persistent slip-bands is a characteristic feature in low stress level fatigue. At some stage during the process, some of these persistent slip-bands degenerate into well definable cracks. When once this happens, the concept of a plastic enclave ahead of such a crack and the subsequent growth of the crack in

terms of the submicroscopic crack nucleation in the enclave is not unreasonable. One may therefore conveniently call the stage up to the creation of the first discernible crack, a nucleation period. An approximate way of estimating this nucleation period is to postulate a continuous decrease in the internal stresses. Now it has been shown by several investigators that fatigue stressing is frequently found to bring in the same microstructural changes as elevated temperature. Processes such as polygonization leading to the formation of subgrains; accelerated precipitation of copper in aluminum 4 percent copper leading to soft spots due to local depletion of solute atoms from the alloy in T-4 condition, have been observed in fatigue at room temperature. These are also processes which, in the absence of fatigue, require an elevated temperature. Therefore, it is proposed that, since polygonization leading to subgrains formation involves a decrease in internal stresses, one may look upon the effect of temperature and the effect of fatigue as being the same insofar as the internal stress σ_i is concerned. That is, we identify σ_i with the initial proportional limit except that, as the fatigue stressing is continued, it continues to decrease locally. This initial proportional limit is considered to be the same as that at which the tangent modulus changes abruptly. It is now suggested that, in the absence of more pertinent information, the decrease of internal stresses due to fatigue may be assumed to follow a relation similar to that due to thermal fluctuations, for which Cottrell and Aytekin[8] give the relation

$$\sigma_i = \sigma_{io} - \frac{RT}{\beta} \log\left(1 + \frac{t}{t_o}\right) \tag{4}$$

where σ_{io} is the initial proportional limit at room temperature, σ_i is the instantaneous value at time t for temperature T; R is the Boltzmann constant, β and t_o are constants. They show under secondary creep conditions (steady state) t_o K_2T where K_2 is a constant. Since in fatigue, cycles to failure is more important, we shall write the above equation in the form

$$\sigma_i = \sigma_{io} - \frac{RT}{\beta} \log\left(1 + \frac{N}{K_2 \omega T}\right) \tag{5}$$

where ω is the frequency. Obviously then, for all applied stress levels below σ_{io}, there will be a nucleation period during which the internal stresses continuously decrease. When the value of the internal stresses becomes smaller than the applied stress, some suitably situated dislocations will leave their sources; regions of fine slip will appear and the degeneration process is initiated. Clearly, the nucleation cycles are given by

$$N_o = K_2 \omega T \left\{ e^{\left(\frac{\sigma_{io} - \sigma}{RT}\right)\beta} - 1 \right\} \tag{6}$$

If the number of cycles to propagation till failure is added to N_o, one will obtain an estimate of the total cycles to failure in low stress level fatigue. With this interpretation of σ_i, the theory is no longer restricted to high stress levels. The stress that formally separates the high and low stress level regime is σ_{io}.

One further modification is necessary before we can write a set of basic equations to describe the fatigue process. This has to do with the crack length term ℓ. It will be discussed in detail elsewhere. It appears that as the crack grows, the finite width of the specimen affects the size of the plastic enclave and the Westergaard solution[9] suggests substituting 2ℓ by $\tan\frac{\pi\ell}{W}$ where 2ℓ is the crack length for a central crack in a sheet of width W. It also appears that the properties of the material can be explicitly brought into the fatigue problem by invoking the Ramberg-Osgood stress strain relation to express the natural plastic stress concentration factor in terms of the theoretical stress concentration factor K_{Tn}. It can be shown[4] that this will give

$$K_n = 1 + \frac{K_{Tn} - 1}{1 + \alpha\left(\frac{K_n \sigma_n}{E}\right)^{\gamma - 1}} \tag{7a}$$

and

$$\epsilon_p = \frac{\sigma_p}{E} + \alpha\left(\frac{\sigma_p}{E}\right)^{\gamma} \tag{7b}$$

$$\sigma_p = K_n \sigma_n \tag{7c}$$

Equation 7a is the Hardrath-Ohman[10] relation and 7b is the Ramberg-Osgood stress-strain relation[11] where ϵ_p is the strain and σ_p is the stress.

We now can write the set of basic equations for fatigue. It must still be noted they do not say anything about the residual stress fields that are created in the plastic enclave when a high stress is followed by a low stress. A theory of fatigue to be complete must give an expression for these stresses and the rate at which they are removed due to subsequent fatigue stressing. With this exception the equations are

$$\frac{1}{C}\frac{d\ell}{dN} = \left(\frac{\sigma_p - \sigma_1}{E}\right)^2 \left(\frac{\sigma_p - \sigma_p'}{\sigma_1}\right)^2 \tan\frac{\pi\ell}{W} \;^{*} \tag{8a-1}$$

$$= \sigma_p^2 \left(\frac{\sigma_p}{\sigma_1} - 1\right)^2 \left(1 - R_1\right)^2 \tan\frac{\pi\ell}{W}\; ; \left(R_1 = \frac{\sigma_p'}{\sigma_p}\right) \tag{8a-2}$$

$$= \sigma_p^2 \left(\frac{\sigma_p}{\sigma_1} - 1\right)^2 \left(1 - \frac{\sigma_m}{\sigma_p}\right)^2 \tan\frac{\pi\ell}{W}\; ; \left(\sigma_m = \frac{\sigma_p + \sigma_p'}{2}\right) \tag{8a-3}$$

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* The effect of stress range is somewhat more complicated than that indicated by equation 8a. It is discussed further in reference 4.

Equations 8a-1, 8a-2, and 8a-3 are alternate forms.

$$\sigma_p = K_n \sigma_n \tag{8b}$$

$$K_n = 1 + \frac{(K_{Tn} - 1)}{1 + \beta\left(\frac{K_n \sigma_n}{E}\right)^{\gamma - 1}} \tag{8c}$$

$$N_o = K_2 \omega T \left\{ e^{\left(\frac{\sigma_{io} - \sigma}{RT}\right)\beta} - 1 \right\} \quad \sigma \leq \sigma_{io} \tag{8d}$$

$$= 0 \qquad\qquad \sigma \geq \sigma_{io}$$

It has been shown elsewhere that[4]

$$K_{Tn} = 2\left(\frac{2\ell}{\rho}\right)^{1/2} \left(1 - \lambda\right)\left(1 + 0.5948\ \lambda^2 - \ldots\right) \tag{8e}$$

when ρ is the crack tip radius and $\lambda = \frac{2\ell}{W}$. Equation 8e is the relation obtained by Isida.

$$\sigma_i = \sigma_{io} - \frac{RT}{\beta} \log\left(1 + \frac{N}{K_2 \omega T}\right) \tag{8f}$$

And finally, we have the limiting condition on the crack growth defined by the relation

$$\left(\frac{\sigma_u}{\sigma_n}\right)^{a_T} = \left(\frac{\ell_{cr}}{\ell}\right) \tag{8g}$$

where ℓ_{cr} is the crack length for catastrophic failure when the net section stress is σ_n. ℓ_o is the length of the maximum crack length for which the critical stress for catastrophic failure is the same as the ultimate tensile strength. a_T is a temperature and material dependent exponent. At room temperature, for brittle materials it has a value 2. As the ductility gradually increases, its value also increases and for infinitely ductile materials it tends to infinity. It will be discussed elsewhere in detail[12]. Equations 8a thru 8e define the crack nucleation and propagation problem. Equation 8f, due to the dependence of the decrease of internal stresses on temperature, enables one to develop a first order theory of fatigue which is valid up to the re-crystallization temperature for materials with monotonically decreasing fatigue life with an increase of temperature. And equation 8g will, when imposed on the solution obtained for crack length versus cycles from the rest of the equations, defines the cycles to failure. Some interesting consequences of these equations will be discussed in references 4 and 12.

III APPROXIMATIONS OF THE GENERAL THEORY FOR PURPOSE OF ACOUSTIC FATIGUE

While the set of equations 8, describe the engineering problem of fatigue in a fairly general manner, they are still too complex for practical engineering purposes. In general, the load inputs are too uncertain (especially in acoustic fatigue) and the inherent scatter too much at the low stress levels characteristic of acoustic fatigue. The problem as formulated above is essentially non-linear. The concept of stress dependent nucleation period below the proportional limit makes the treatment based on equivalent damage concept at different stresses somewhat more difficult to apply. However, since the rate of growth of fatigue cracks at low stress levels is extremely small, the reduction in residual strength is negligible for a considerable part of the fatigue cycling. Therefore simplified forms of equations 8, if they give reasonable comparison between theory and test, are desirable to describe the acoustic fatigue problem. Therefore, it appears that it is desirable to keep the above equations in mind and try to obtain simplified expressions which will be adequate for the problem under consideration.

The basic equation for crack propagation, equation 8 is

$$\frac{1}{C}\frac{d\ell}{dN} = \left(1-R_1\right)^2\left(\sigma_n K_n\right)^2\left\{\frac{\sigma_n K_n}{\sigma_i} - 1\right\}^2 \tan\frac{\pi\ell}{W} \tag{9a}$$

For the sake of simplicity, we shall assume that $\tan\frac{\pi\ell}{W} \approx \frac{\pi\ell}{W}$. Test observations show that for a material like 2024 T3, the endurance limit is about 20,000 psi, the yield point is about 50,000 psi and the ultimate tensile strength is about 72,000 psi. Before failure, the plastic stress in the enclave cannot exceed the ultimate tensile strength. Therefore, if the applied stress is close to the endurance limit, the natural stress concentration factor cannot be much larger than 3.5. For stress close to the yield point (ii) in the high stress level fatigue, the value of K_n will be closer to 1. Equation 9a can be written for engineering purposes in the form

$$\frac{1}{K_n C}\frac{d\ell}{dN} = \left(1-R_1\right)^2\sigma_n^2\left\{\frac{\sigma_n}{\sigma_i/K_n} - 1\right\}^2\ell \tag{9b}$$

Since K_n does not vary much beyond 1.5 to 3.5, the results of assuming K_n to be approximately a constant will be well within the scatter band of fatigue, especially in low stress level fatigue. Assuming that the test stress ratio R_1 -1 and absorbing it into the constant (along with K_n), we may write therefore

$$\frac{1}{C}\log\frac{\ell}{\ell_o} = \sigma_n^2\left\{\frac{\sigma_n}{\sigma_i/K_n} - 1\right\}^2\left(N - N_o\right) \tag{10}$$

In the high stress level regime N_o 0. The limiting condition on the crack growth is

$$\log \frac{\ell}{\ell_0} = \log \left(\frac{\sigma_u}{\sigma}\right)^a \tag{11}$$

Therefore, the cycles to failure are

$$N = C_1 \frac{\log \left(\frac{\sigma_u}{\sigma_n}\right)^a}{\sigma_n^2 \left\{ \frac{\sigma_n}{\sigma_i / K_n} - 1 \right\}^2} \tag{12}$$

In the high stress level regime, the critical crack length is generally small compared with the specimen width, so that we may safely assume

$$\sigma_n \approx \sigma \tag{13}$$

where σ is the gross section stress and thus obtain

$$N = C_1 \frac{\log \left(\frac{\sigma_u}{\sigma}\right)^a}{\sigma^2 \left\{ \frac{\sigma}{\sigma_i / K_n} - 1 \right\}^2} \; ; \; \left(C_1 = \frac{1}{C}\right) \tag{14}$$

Test data provided by Illg[13] was used to evaluate the accuracy with which equation 14 describes the test results for assumed values of $K_n = 1$, 2 and 3. For a chosen value of K_n, constant C_1 is determined by taking a value of σ and the corresponding cycles to failure as determined by tests. The exponent "a" was found from test results to be about 5. Table I gives the test data of Illg. σ_i was assumed to be 18,000 psi. Table II gives the value of C_1 as determined from various test points, the chosen stress varying from 70,000 psi to a low of 23,000 psi. It will be recalled that the high stress level regime in the scheme presented here is above 34,000 psi, this being the value of the proportional limit as defined here. The cycles to failure in Table II was determined on the basis of the average value of C_1. The first observation that one can make from the table is that for any given value of K_n the value of C_1 is approximately a constant in the sense that the theoretical results thus determined will be well within the scatter band. Thus for $K_n = 1$, the value of C_1 ranged from a low of 1.3×10^{13} to a high of 5.6×10^{13} with an average value of about 3.4×10^{13}. In practical engineering problems the stresses used for tests may be expected to be between 30,000 to 40,000 psi. In this region, the value of C_1 varies from 2.24×10^{13} to 4.85×10^{13}, the range of scatter from the average being relatively small as far as fatigue results go. It will also be noted that $K_n = 1$ should be strictly applicable to the case where the applied stresses are close to the yield point. Therefore, one should in principle expect a good comparison between theoretical values and test results in this range and considerable divergence at the low stress levels. As the value of K_n continues to increase, the comparison between

TABLE I

FATIGUE TEST DATA OF ILLG FROM NACA TN 3866[13]

Stress psi	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{ave}
72,500	1/4									
72,000	7	7								7
70,000	102	104	131							112
65,000	342	663	967							657
55,000	3,000	6,000	8,000							5,700
50,000	10,000	11,000	16,000							12,000
45,000	11,700	16,000	31,000	36,000	51,000					29,000
40,000	37,000	39,000	60,000	68,000	70,000	87,000				60,000
35,000	40,000	66,000	109,000	161,000						94,000
30,000	119,000	185,000	241,000	277,000	283,000	339,000				241,000
25,000	205,000	349,000	1.197×10^6	1.483×10^6						809,000
23,000	645,000	1.404×10^6	2.07×10^6	3.33×10^6						1.86×10^6
20,000	550,000	880,000	3.5×10^6	4.5×10^6	6.4×10^6	1[illegible] 10^6	13.2×10^6	2.3×10^6	84.9×10^6	14.9×10^6
18,000		$>100 \times 10^6$								

TABLE II

VALUES OF $C_{1_{ave}}$ & N AS DETERMINED FROM TEST DATA

$$N = C \frac{\log\left(\frac{\sigma_u}{\sigma}\right)^{a}}{\sigma^2\left\{\frac{\sigma}{\sigma_i/K_n} - 1\right\}^2}$$

		$K_n = 1$		$K_n = 2$		$K_n = 3$			
Stress psi	$N_{exptl.}$	$C\times10^{-13}$	N_{th} for C_{ave}	$C\times10^{-14}$	N_{th} for C_{ave}	$C\times10^{-14}$	N_{th} for C_{ave}	$N_{exptl.}$ Min.	$N_{exptl.}$ Max.
70,000	110	2.62	143	1.44	230	3.6	255	102	131
65,000	660	3.50	640	2.00	990	5.0	1,100	342	967
55,000	5,700	5.26	3,700	3.24	5,300	8.21	5,800	3,000	8,000
50,000	12,000	4.25	9,600	2.77	13,000	7.25	13,900	10,000	16,000
45,000	29,000	5.55	17,700	3.95	22,200	10.5	23,300	11,700	51,000
40,000	60,000	4.83	42,[illegible]00	3.86	46,700	10.4	48,500	37,000	87,000
35,000	94,000	2.85	112,000	2.63	107,000	7.5	106,000	40,000	161,000
30,000	240,000	2.16	375,000	2.67	272,000	7.8	256,000	119,000	339,000
25,000	809,000	1.53	1.79×10^{6}	3.20	760,000	10.2	670,000	205,000	1.5×10^{6}
23,000	1.86×10^{6}	1.33	4.75×10^{6}	4.16	1.35×10^{6}	13.75	1.14×10^{6}	645,000	3.3×10^{6}
C_{ave}		3.39		3.0		8.4			

theory and test results should gradually become worse in the high stress level region and better at the low stress levels. This is essentially what we find from Table II. But in all the cases, as the plotting of the data in Table II on Figure 2 shows, the theoretical curves are well within the scatter range of the tests and the plotted curves alone will not be able to reveal a relative choice between the various values of K_n. An examination of the table, however, shows that the value of K_n 2 gives the best overall fit for this data. In the absence of contradictory information, we may therefore choose equation 14 as a reasonable approximation for engineering purposes, with a value of K_n of about 2. For purposes of acoustic fatigue where the stress levels are closer to the endurance limit and the high stress levels are not very important from the standpoint of contribution to damage, we may even assume K_n to be between 3 and 4. In other words, we shall use K_n as a floating constant of the problem, used to obtain a good comparison between theory and test. Another extremely important conclusion we draw is the following one: Equation 14 was derived on the basis of high stress level fatigue, there being no nucleation period. After obtaining the equation, we applied without discrimination to the low stress level regime also and find that the theoretical expression predicts the data reasonably well in this region also. We shall therefore, as a matter of engineering simplicity, assume that equation 14 is valid over the whole range and by implication therefore, that

$$\frac{1}{C} \log \frac{\ell}{\ell_o} = \left(1 - R\right)^2 \sigma^2 \left\{ \frac{\sigma}{\sigma_i / K_n} - 1 \right\}^2 N \tag{15a}$$

is equally valid. By so doing, we are essentially assuming that the residual strength continues to decrease from the first cycle in the low stress level region also. Actually, it is clear that this is not true. But the extent of decrease in the strength in the low stress level region is so small in the initial stages, (as shown for example in Figure 3, reproduced from reference 2) that for practical purposes it is approximately constant. This assumption involves considerable simplification and hence we shall adopt equation 15a or its alternate forms as appropriate for the treatment of the acoustic fatigue problem.

Since the concept of identifying σ_i with the endurance limit is reasonable only for fully reversed loading, it needs modification where a mean stress exists. Not enough thought has been given to the underlying basic problem in this case. Without further discussion we shall assume in line with current practice, that the following relation holds

$$\sigma_{mi} = \sigma_i \left\{ 1 - \left(\frac{\sigma_m}{\sigma} \right)^g \right\} \tag{15b}$$

where σ_{mi} is the effective endurance limit in the presence of a mean stress σ_m. The equation, with the value of g equal to 1 is associated with the name of Goodman; with g equal to 2 with the name of Gerber. Equations 15a and 15b, together with the limiting condition

$$\log \frac{\ell}{\ell_o} = \log \left(\frac{\sigma_u}{\sigma} \right)^a$$

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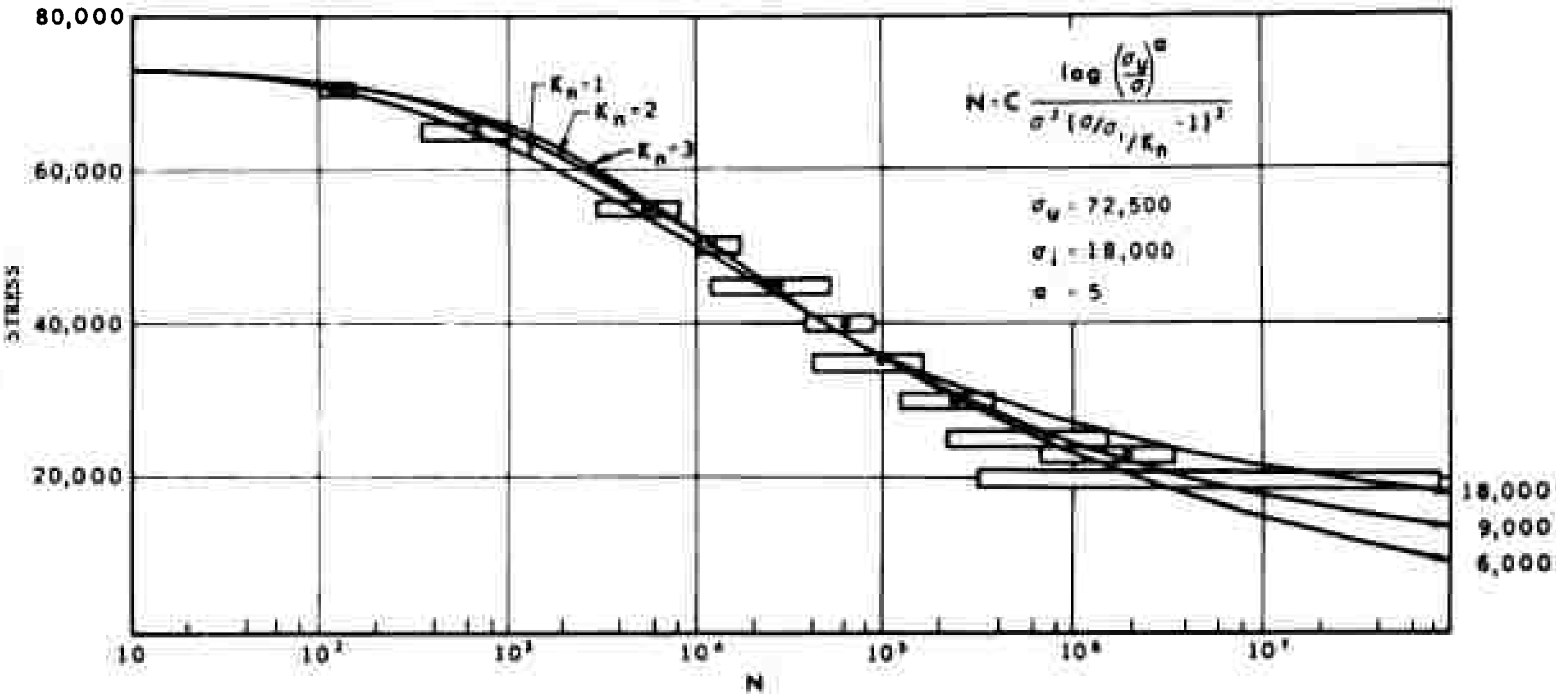

FIGURE 2 COMPARISON BETWEEN THEORETICAL CURVES AND TEST DATA

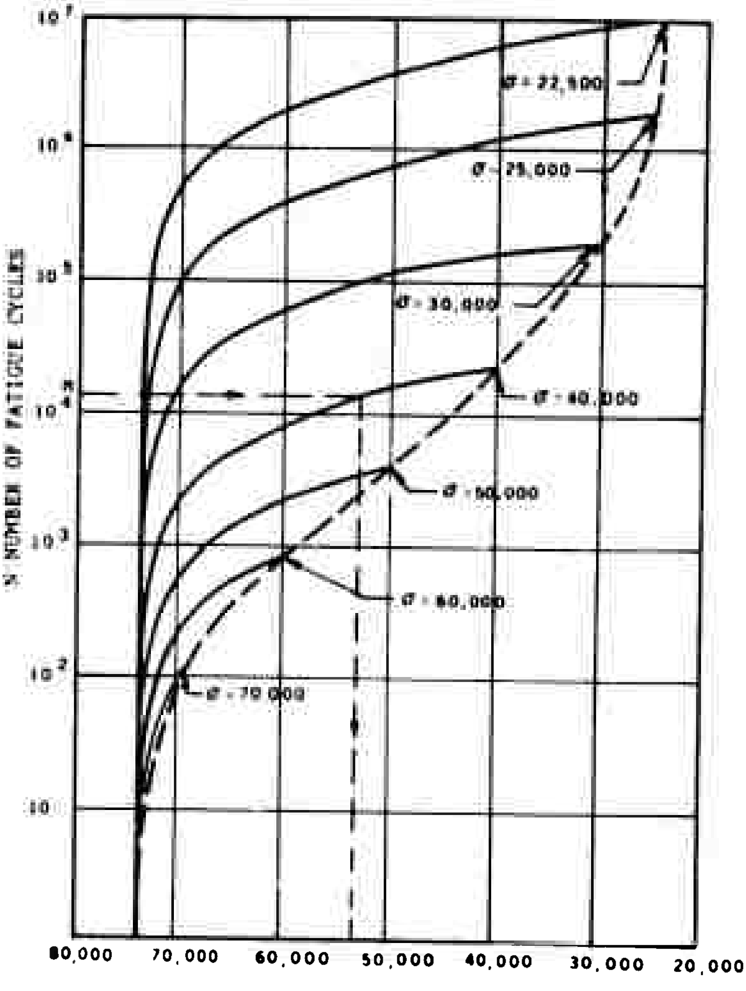

FIGURE 3 ESTIMATION OF RESIDUAL STATIC STRENGTH DURING FATIGUE (LINEAR STRESS SCALE)

are sufficient to give a reasonable and sufficiently approximate treatment of the acoustic fatigue problem.

IV CONCEPT OF EQUIVALENT DAMAGE AT DIFFERENT STRESS LEVELS AND APPLICATION TO RANDOM LOADING

For purposes of acoustic fatigue it is convenient to rewrite equation 15a in the alternate form

$$\frac{1}{C}\log\frac{\ell}{\ell_o} = \sigma^2\left(\frac{\sigma}{\sigma_i/K_n} - 1\right)^2\left(1 - \frac{\sigma_m}{\sigma}\right)^2 N \tag{16}$$

where σ_m is the mean stress $\frac{1}{2}(\sigma + \sigma')$. It includes the mean stress due to the acoustically induced vibration as well as the inplane stress for which the panel is designed. This is schematically shown in Figure 4. The inplane static stress is of considerable importance in determining the limiting conditions to failure, as we shall presently see. In principle, if curves of equation 16 are available, we will be in a position to define the concept of equivalent damage. The only unknown in equation 16 is C, which can be determined from equation 14. We are now in a position to define the concept of equivalent damage. The equivalent damage concept relates the cycles of applied stress at two different stress levels to cause the same amount of damage in terms of the length of the dominant crack. This may be seen from Figure 5, where a schematic presentation of the crack growth for two stress levels σ_1 and σ_2 $(\sigma_1 < \sigma_2)$ is given. If n cycles are applied at stress σ_1 initially, they would cause the dominant crack to grow to a length ℓ_1/ℓ_o. The crack will reach the same length by the application of n_2 cycles of stress σ_2 as shown. The condition for the same amount of damage in terms of the same length of crack is clearly

$$\sigma_1^2\left\{\frac{\sigma_1}{\sigma_i/K_n} - 1\right\}^2\left\{1 - \frac{\sigma_{m_1}}{\sigma_1}\right\}^2 n = \frac{1}{C}\log\frac{\ell}{\ell_o} = \sigma_2^2\left\{\frac{\sigma_2}{\sigma_i/K_n} - 1\right\}^2\left\{1 - \frac{\sigma_{m_2}}{\sigma_2}\right\}^2 n_2 \tag{17}$$

Therefore, if the damage is accruing at different stress levels, we can convert all the damage to a specific reference stress level and inquire into the limiting conditions of crack growth at the reference stress level to cause catastrophic failure.

In acoustic fatigue, which is in a certain sense a special case of random loading one fundamental difficulty is the fact that we really do not know the number of cycles n_q associated with a specific stress σ_q except on a probabilistic basis. The result is an uncertainty in the crack length at any stage. To see the method of solution a little better, let us assume that the frequency distribution curve for the applied stress is available. This, for example, may be the frequency distribution curve associated with the rms stress. This is shown schematically on the right hand side in Figure 6 with the stress on the ordinate and the probability $P(\sigma)$ plotted on the abscissa. Now suppose n cycles are applied in a random manner. If all the stress pulses are applied at the highest stress level, the crack would presumably be of

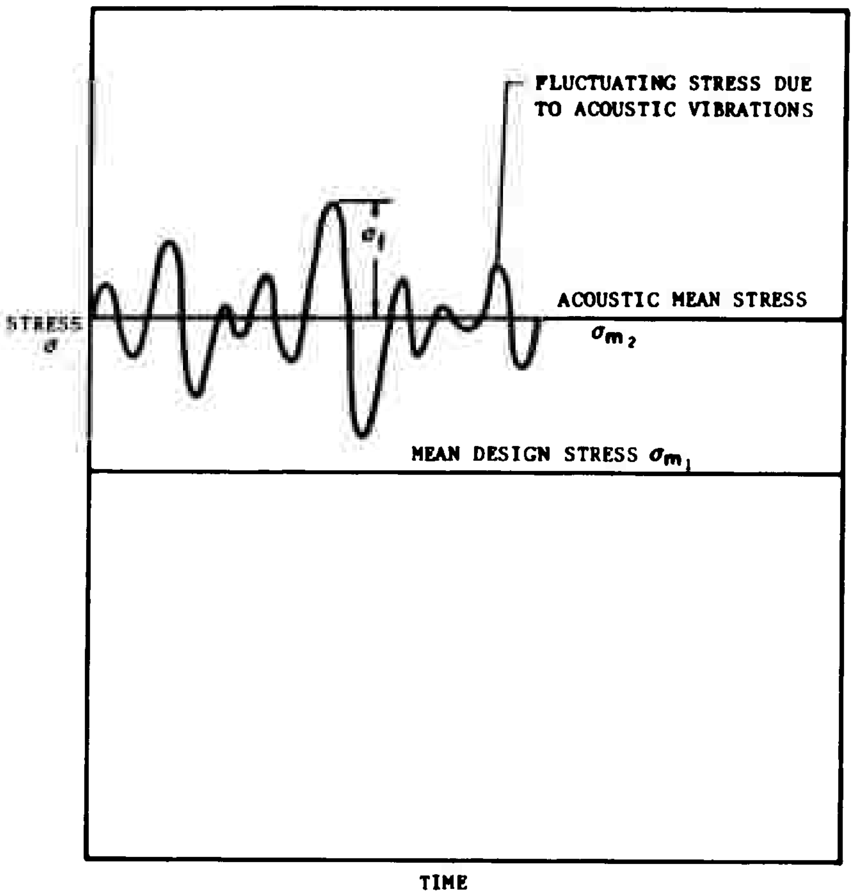

FIGURE 4 SCHEMATIC OF THE STRESS IN THE PANEL

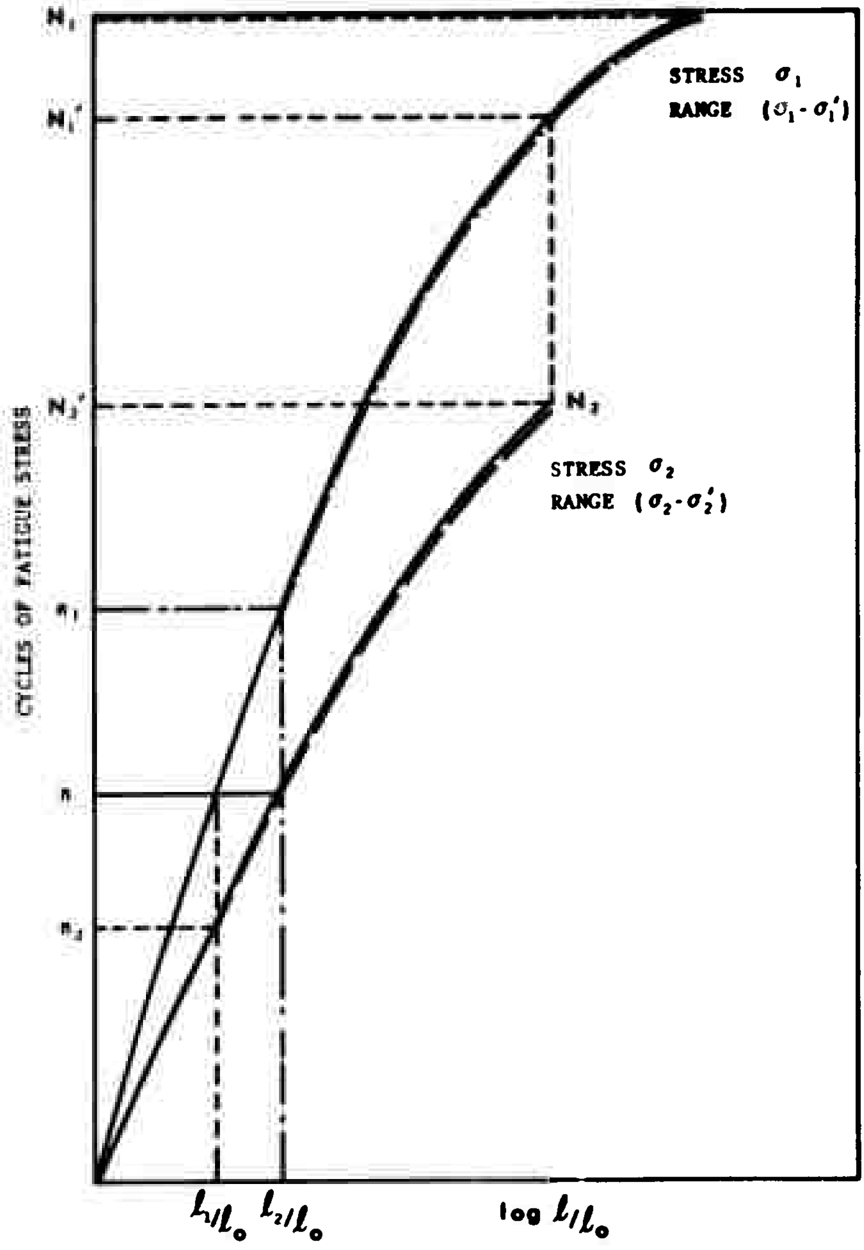

FIGURE 5 SCHEMATIC FOR DISCUSSION OF CUMULATIVE DAMAGE

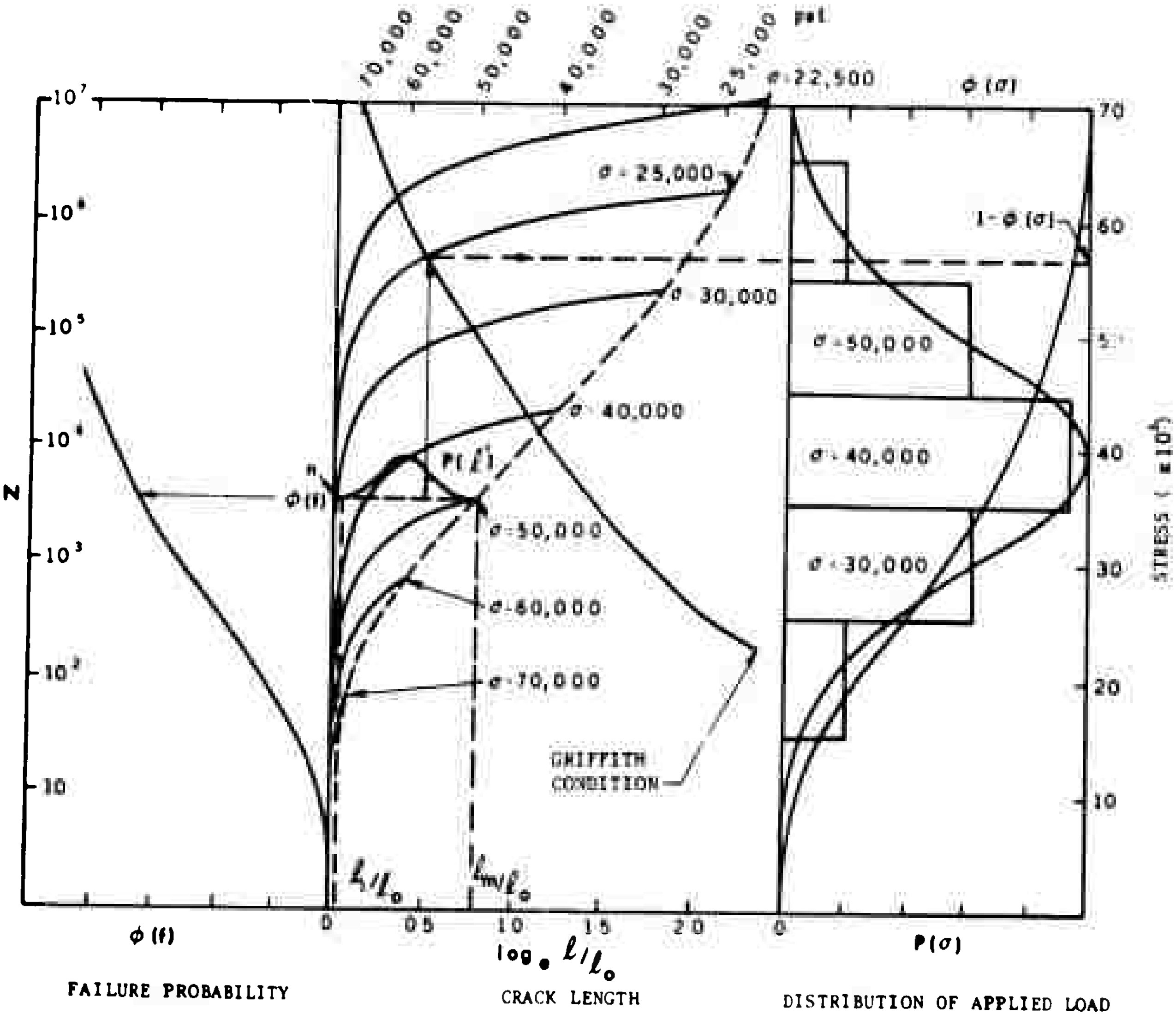

FIGURE 6 SCHEMATIC REPRESENTATION FOR THE DISCUSSION OF RANDOM LOADING IN FATIGUE

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length ℓ_m/ℓ_o and if all of them are applied at the lowest stress level, the crack would be of length ℓ_1/ℓ_o as shown. But since the stress pulses are applied in a random manner, one can only talk of the probability of the crack length being of a certain value. Therefore, one can in principle draw a frequency distribution curve for the crack length which is, of course, related to the frequency distribution curve of the applied stress pulses. Let the probability of the crack length being ℓ be R(ℓ'). Whether failure would occur on the application of the next pulse is dependent upon whether or not the stress pulse is either equal to or larger than that necessary for the catastrophic propagation of the crack whose length is ℓ'. The magnitude of this stress is given by the limiting condition which is assumed to be the Griffith condition and is plotted as shown. The probability that a stress pulse of this magnitude or larger would occur is given by $\{1 - \phi(\sigma)\}$ where $\phi(\sigma)$ is the cumulative distribution of σ. The probability that failure would occur now is the product of these two probabilities, $P(\ell')\ \{1 - \phi(\sigma)\}$. But since the crack can be anywhere between the two extremes indicated, it is apparent that the probability that failure would occur after application of n stress cycles is equal to the integrated probability over the crack length between the two expected limits. Having obtained this probability, one could plot $\phi(f)$ and thus obtain a cumulative distribution function as shown schematically for failure of the panel at various stages of the applied stress cycles whether due to a gust spectra, or a maneuver spectra or an acoustic spectra.

The main question we face at this stage is "to what extent are we justified in trying to treat the problem of acoustic fatigue in such detailed manner?". While we know that the loading is random, there is considerable doubt about the estimated load magnitudes. Secondly, the frequency of stress pulses in acoustic fatigue is of the same order as the frequency of the fundamental mode of vibration and typically this is about a few hundred cycles per second. Therefore, if the chosen unit time interval is not too small, one may assume safely for analytical simplicity, that within the unit time limit under consideration, the probable number of cycles at the different stress levels are in fact applied. In other words, we assume that the associated probability for a particular crack length after n number of random cycles is 1. The fatigue failure of the panel itself is still probabilistic, as the condition for catastrophic failure (Griffith condition or equivalent) is still stress dependent. It is clear that the probability of failure is now simply $\{1 - \phi(\sigma)\}$.

V SOLUTION TO THE PROBLEM OF ACOUSTIC FATIGUE

Equation 17 can be written in the general form

$$\sigma_q^2 \left\{ \frac{\sigma_q}{\sigma_{mi}/K_n} - 1 \right\}^2 \left\{ 1 - \frac{\sigma_{mq}}{\sigma_q} \right\}^2 \Delta n_q = \sigma_r^2 \left\{ \frac{\sigma_r}{\sigma_{mi}/K_n} - 1 \right\}^2 \left\{ 1 - \frac{\sigma_{mr}}{\sigma_r} \right\}^2 \left(\Delta n_{rq} \right) \qquad (18)$$

where Δn_q is the number of cycles applied at stress σ_q, σ_{mq} is the associate mean stress; σ_{mi} is the endurance limit when the mean stress is σ_{mq}; (Δn_{rq}) is the equivalent number of cycles at the reference stress σ_r, mean stress σ_{mr} and endurance limit σ_{mi}. Since the reference stress is arbitrary we can choose σ_{mr} σ_{mq} σ_m. This is a justifiable assumption from other considerations also. It has already been stated that the panel is designed to carry a constant design stress which is essentially a mean stress for the problem; and a component of acoustically induced vibrations which is approximately a constant in the sense that it has a long wave length and does not vary sensibly. Furthermore, one may expect in general, that the acoustic component of the mean stress is considerably smaller than the static design stress on the panel. Therefore, without loss of generality, we may assume that the mean stress is essentially constant for all stress amplitudes σ_q including σ_r. Therefore, we can write

$$\left(\Delta n_{rq} \right) = \left\{ \frac{\sigma_q}{\sigma_r} \, \frac{\left(\sigma_q/\sigma_{mi}/K_n - 1 \right) \left(1 - \frac{\sigma_m}{\sigma_q} \right)}{\left(\sigma_r/\sigma_{mi}/K_n - 1 \right) \left(1 - \frac{\sigma_m}{\sigma_r} \right)} \right\}^2 \Delta n_q \qquad (19)$$

In particular, if Δn_q is equal to 1, the right hand side of the equation gives the fractional number of cycles at the reference stress to cause the same damage as one cycle of stress at the state defined by (σ_q , σ_m).

At this stage it is desirable to briefly outline the method of approach that will be developed here. In a certain sense, equation 19 is the fundamental equation for discussing the acoustic fatigue problem. The Rayleigh distribution function is converted to an appropriate histogram with stress intervals $\Delta\sigma_q$ starting from zero. We first focus our attention at an rms stress peak and its associated frequency. The damage sustained by pulses at the various stress levels corresponding to the median points of the stress intervals are converted into an equivalent number of stress pulses at a reference stress level by the use of equation 19. Since one characteristic feature of acoustic fatigue is the application of stress pulses simultaneously at various frequencies, it is more convenient to think in terms of time to failure rather than cycles to failure. Accordingly the equivalent number of cycles at the reference frequency is converted to an equivalent time for the same damage at the reference frequency. This process is repeated for all significant rms stress peaks at the different frequencies. Since all the damage is now referred to a reference stress and frequency, these various equivalent times can be added together. The problem of acoustic fatigue is now reduced to one discussing the fatigue failure at one stress and frequency and the probability of failure at that stress.

Let us consider the frequency ω_q at which the rms stress peak $\bar{\sigma}_q$ occurs. The frequency distribution function for the amplitude peaks associated with this rms stress is

$$P\left(\sigma_q\right) = \left(\frac{\sigma_q}{\bar{\sigma}_q}\right) \exp^{-\frac{1}{2}\left(\frac{\sigma_q}{\bar{\sigma}_q}\right)^2} \tag{20}$$

In unit time the probable number of stress pulses Δn_q is therefore

$$\Delta n_q = \omega_q \left(\frac{\sigma_q}{\bar{\sigma}_q}\right) \exp^{-\frac{1}{2}\left(\frac{\sigma_q}{\bar{\sigma}_q}\right)^2} d\left(\frac{\sigma_q}{\bar{\sigma}_q}\right) \tag{21}$$

As stated earlier we shall assume that this number of stress pulses are actually applied. The total equivalent number of cycles at the reference stress, corresponding to the rms stress $\bar{\sigma}_q$, acting per unit time is therefore

$$\Delta n_{erq} = \int_{\sigma_{mi}/K_n}^{\alpha} \omega_q \left(\frac{\sigma_q}{\bar{\sigma}_q}\right) \exp^{-\frac{1}{2}\left(\frac{\sigma_q}{\bar{\sigma}_q}\right)^2} \left\{ \left(\frac{\sigma_q}{\bar{\sigma}_q}\right) \frac{\left(\frac{\sigma_q}{\sigma_{mi}/K_n} - 1\right)\left(1 - \frac{\sigma_m}{\sigma_q}\right)}{\left(\frac{\sigma_r}{\sigma_{mi}/K_n} - 1\right)\left(1 - \frac{\sigma_m}{\sigma_r}\right)} \right\}^2 d\left(\frac{\sigma_q}{\bar{\sigma}_r}\right) \tag{22}$$

The lower limit of integration σ_{mi}/K_n is chosen on the basis that below an amplitude of vibration σ_{mi}/K_n, acoustic fatigue will not take place for any predefined, arbitrarily large period of exposure to the acoustic field. If the reference frequency is ω_r, we can define an equivalent, non-dimensional time at the reference state corresponding to the state "q" by the condition

$$t_{erq} = \frac{\Delta n_{erq}}{\omega_r} \tag{23}$$

It may be noted that Δn_{erq}, and hence t_{erq}, will still be a function of ω_q and $\bar{\sigma}_q$. Thus the integration process must be repeated at the different σ_q's occurring at the different frequencies and the non-dimensional equivalent times thus obtained must be added together. Thus

$$t_{er} = \sum_{q=1}^{q} t_{erq} \tag{24}$$

where q 1, 2, .. q denote the significant rms stress peaks. In actual practice one may use the highest rms stress peak for a first approximation and depending upon the accuracy, take into account the other stress peaks occurring at the other frequencies also in subsequent approximations. It will be noted that since the t_{erq}'s are computed for unit actual time for the same damage, t_{er} is essentially a weighted sum of unit non-dimensional time at the reference state to cause the same damage as the damage caused by the simultaneous application of stresses $\bar{\sigma}_q$ at frequencies ω_q.

Now consider a stress pulse of amplitude $\sigma_{q\ell}$. The length of the crack that would suffer catastrophic failure at this stress is

$$\log \frac{\ell}{\ell_o} = \log \left(\frac{\sigma_u}{\sigma_{q\ell}} \right)^a$$

It may be shown that the number of cycles it would take at the reference stress to grow a crack which would be just critical for catastrophic failure at the stress $\sigma_{q\ell}$ is

$$n_r = \frac{1}{C} \frac{\log \left(\frac{\sigma_u}{\sigma_{q\ell}} \right)^a}{\left\{ \sigma_r \left(\frac{\sigma_r}{\sigma_{mi}/K_n} - 1 \right) \left(1 - \frac{\sigma_m}{\sigma_r} \right) \right\}^2} \tag{25}$$

That is, $\sigma_{q\ell}$ is the residual strength of the specimen after the application of n_r cycles at the stress σ_r. Obviously, at the reference frequency it will take time

$$t_r = \frac{n_r}{\omega_r} \tag{26}$$

to reach the residual strength $\sigma_{q\ell}$. Therefore, if at time t_r, a . . stress pulse $\sigma_{q\ell}$ or larger is applied, catastrophic failure would occur. In acoustic fatigue however, the damage is "simultaneously" being caused at various frequencies. Equation 24, gives the weighted non-dimensional time to cause the same amount of damage at the reference state, as the sum of individual damages at the different frequencies and stresses. Therefore, the actual time at which the residual strength σ_q is reached is clearly given by

$$t_A = \frac{t_r}{t_{er}} \tag{27}$$

But in acoustic fatigue there is a probability associated with the application of the stress pulse $\sigma_{q\ell}$. Within the limits of the assumptions made here, the probability of failure at time t_A is precisely the same as

$$P(f) = \left\{ 1 - \phi\left(\sigma_{q\ell}\right) \right\} \qquad (28)$$

where $\phi(\sigma_{q\ell})$ is the cumulative probability of the distribution function for the rms stress $\bar{\sigma}_q$.

The solution to the problem of acoustic fatigue as formulated here is now evident. Given a rms stress versus frequency curve, we compute the t_A, with the help of equations 22 through 27 for various assumed values of σ_q ranging from σ_u to σ_{mi}/K_n. The probability of failure at t_A is given by equation 28. Therefore, we can plot a curve of t_A versus P(f) for the given rms stress versus frequency curve. This is repeated for different rms stresses versus frequency curves. One will thus obtain curves of rms stress versus cycles to failure and the associated probabilities. By joining the points of equal probability at the various rms stress levels, one will then obtain curves of rms stress versus cycles to failure with probability of failure as parameter for each curve.

There is one point which is worth noting here. In a typical case the rms stress versus frequency curve has more than one peak and when we estimate the cycles to failure and want to plot it, the question arises "what is the most reasonable way of representing this input in a fatigue curve"? If there is only a single peak one can use the magnitude of this peak as representative of the stress response without any ambiguity. If there is more than one peak, it is not immediately certain that as the acoustic input intensity is increased, the rms stress peaks at the various frequencies increase proportionately. Observations are apparently available which indicate that as the glancing angle of jet on the panel is changed, the rms stress versus frequency curve tend to change. There are at present no clearly definable methods available to represent the rms stress versus frequency by a single parameter for presentation of acoustic fatigue data. As a first approximation however, one may assume that the magnitude of the largest rms stress in an rms stress versus frequency curve and its variation as the acoustic input changes may be used as a parameter for presentation of acoustic fatigue data.

A further comment is in order regarding the estimation of t_r. It will be noted that the reference stress and frequency are arbitrary. As a matter of convenience, we can therefore choose the frequency of the fundamental mode of vibration as the reference frequency and for various amplitudes of vibration ($\sigma_{q\ell}$ $\sigma_{r\ell}$) determine from actual tests, the time to failure t_r. Using the same reference frequency, and a reference stress, we can compute from t_{er}, the weighted equivalent non-dimensional time as discussed above. Using these two terms to determine the actual time to failure t_A is better. In a certain sense, the curves of $\sigma_{r\ell}$ vs t_r form the base data with which we now try to estimate the time to failure in acoustic fatigue problem. The problem is analogous to having basic fatigue data in the term of σ vs N curves and then trying to predict cumulative damage with the help of this data and theories such as Miner's, or variations thereof.

VI IMPORTANCE OF MEAN STRESS ON THE RESULTS

It was mentioned earlier that in the acoustic fatigue problem the mean stress plays a very important part in determining the failure conditions, both in terms of time to failure and the associated probability to failure. This may be seen in the following manner. In acoustic fatigue while the damage itself is occurring at relatively low stresses, the failure at any instant is dependent upon the application of high enough stress levels, that is to say that the extent of damage in terms of crack growth is controlled by the low stress end of the frequency distribution curve and the failure by the high stress end, (i.e., the tail of the distribution curve). The condition for failure is

$$\frac{\ell_{cr}}{\ell_o} = \left(\frac{\sigma_u}{\sigma}\right)^a$$

The stress σ consists, as shown in Figure 4, of three components σ_{m_1}, σ_{m_2} and σ_f where σ_{m_1} is the design mean stress on the panel, which is typically of the order of about 20,000 psi, σ_{m_2} is the mean stress associated with the acoustically induced vibrations and σ_f is the amplitude of the fluctuating component of the acoustically induced vibration. σ_{m_2} may be expected to be about 1,000 psi and σ_f may have an rms value of about 5,000 psi. The probability of occurrence for high amplitudes dies off rather quickly and so one may expect the fluctuating stress not to exceed much beyond 20,000 to 30,000 psi. So in the presence of the design mean s ress, the peak amplitude may be expected to be about 45,000 to 55,000 psi sometime or other and in its absence about 25,000 to 35,000 psi. Let us assume the two are 50,000 and 30,000 psi respectively. For a material like 2024-T3, σ_u is about 70,000 psi and "a" has a value of 5. Thus in the presence of the design mean stress σ_{m_1}.

$$\frac{\ell_{cr}}{\ell_o} = \left(\frac{70000}{50000}\right)^5 \approx 5.4$$

In the absence of the design mean stress

$$\frac{\ell_{cr}}{\ell_o} = \left(\frac{70000}{30000}\right)^5 \approx 69.1$$

Therefore, the critical crack length to failure in the presence of the design mean stress σ_{m_1} may be expected to be an order of magnitude smaller. The cycles (or time) to failure which is dependent upon the logarithm of ℓ_{cr}/ℓ_o may correspondingly be expected to be significantly different though not as much. What is more serious is the associated probability of failure. Suppose in a test simulation, only the acoustic inputs are simulated and no consideration is given to the simulation of design stress. For a particular crack length, suppose a stress σ is needed to cause catastrophic failure. The probability that this would occur is clearly

$$P_1(f) = \left\{ 1 - \phi(\sigma) \right\} \qquad (29)$$

Remembering that the failure is determined by the tail end of the distribution function, it will be noted that $P(\sigma)$ is small, the larger the value of σ. However, if σ is assumed to consist of a mean stress σ_m which is constant, a fluctuating stress of $(\sigma - \sigma_m)$ is needed to cause failure. The probability of failure in this case will therefore be

$$P_2(f) = 1 - \phi\left(\sigma - \sigma_m\right) \tag{30}$$

$P_2(f)$ is obviously larger than $P_1(f)$. The effect of the mean stress is to effectively shift the sign to the left of the distribution curve by the amount σ_m. Therefore, for the same fluctuating stress, the effect of superimposing a mean stress is to increase the probability of failure of any instant.

To summarize, the influence of the design mean stress on the time to failure in acoustic fatigue is considerable and affects the result in two ways. In the first instance, it decreases the effective crack length for catastrophic failure. Rough estimates indicate an order of magnitude difference in the crack length. Secondly, the probability of failure may be increased substantially by the presence of mean stress. Somewhat compensating these two effects is the effect of a tensile design mean stress. A tensile stress in the panel tends to increase the frequency of fundamental mode and somewhat decrease the stress peak amplitudes. This clearly increases the fatigue life for the same acoustic input. To theoretically estimate the compensating effect is very involved but there is little doubt that it exists. Any test simulation of acoustic fatigue on typical panels without simulation of the design mean stress may therefore be expected to be highly erroneous, if not useless, insofar as it is used to make estimates of actual failures in practice.

VII EFFECT OF ACOUSTIC FATIGUE ON PROGRESSIVE LOSS OF STIFFNESS

Let us consider the case of a thin cylindrical shell. The critical buckling strength of such a shell is generally found to obey the relation

$$\sigma_{cr} = \frac{E}{\sqrt{3\left(1-\mu^2\right)}}\left(\frac{t}{R}\right) \qquad (31)$$

where μ is Poisson's ratio; t is the shell thickness and R is the shell radius. One of the results of acoustic fatigue was shown to be the growth of a crack. Suppose a shell is subjected to an axial load which is less than the critical buckling load. It will be interesting to study the extent to which the critical load is reduced by the growth of the crack. A literature survey indicates that there is no published data on this problem. The equation 31 above contains no term that can reflect the effect of a growing crack. Similarly for buckling of a long cylinder under uniform external pressure one obtains

$$\sigma_{cr} = \frac{1}{4\sqrt{1-\mu^2}}\,E\left(\frac{t}{R}\right)^3 \qquad (32)$$

This equation also does not contain any term that may conceivably reflect the effect of a crack. In order to obtain some understanding of the problem, tests are performed on a 2 inch radius mylar cylinder of .010 inch thickness and 9 inches long. Its buckling strength was determined in the absence of any crack and general buckling was found to occur at a load of 54 x 2.36 ≈ 127.4 pounds. A circumferential cut is now made into the cylinder and at ½ inch intervals of the cut, the buckling strength is determined. The results are plotted as shown in Figure 7. The circles indicate the buckling strength for various lengths of the cut and the three square points indicate the first wave formation. A tendency has been found for the preferential formation of the first wave near the tips of the cut. In each case, when the load was removed after general buckling, no permanent deformation was noticed. It will be noted from the plot of the test data, that no significant reduction in the buckling strength took place until the length of the cut approached almost 2½ inches. Considering the fact that the radius of the cylinder is only 2 inches, this is a fairly large percentage of the circumference, in fact almost 40 percent. On the basis of this information it seems reasonable to assume that reduction in the residual strength is more critical in cracks generated due to acoustic fatigue than reduction in stiffness and the corresponding decrease in buckling strength.

Another form of crack propagation which may conceivably affect the stiffness is the formation of innumerable submicroscopic cracks. This is not considered serious at all from the standpoint of reduction in stiffness, since, when a compressive load is applied, the crack essentially collapses and the compressive load is transmitted relatively freely through the crack region. However, it is desirable to investigate experimentally these two aspects before we conclude that reduction in residual static strength is a more critical condition than reduction in buckling strength.

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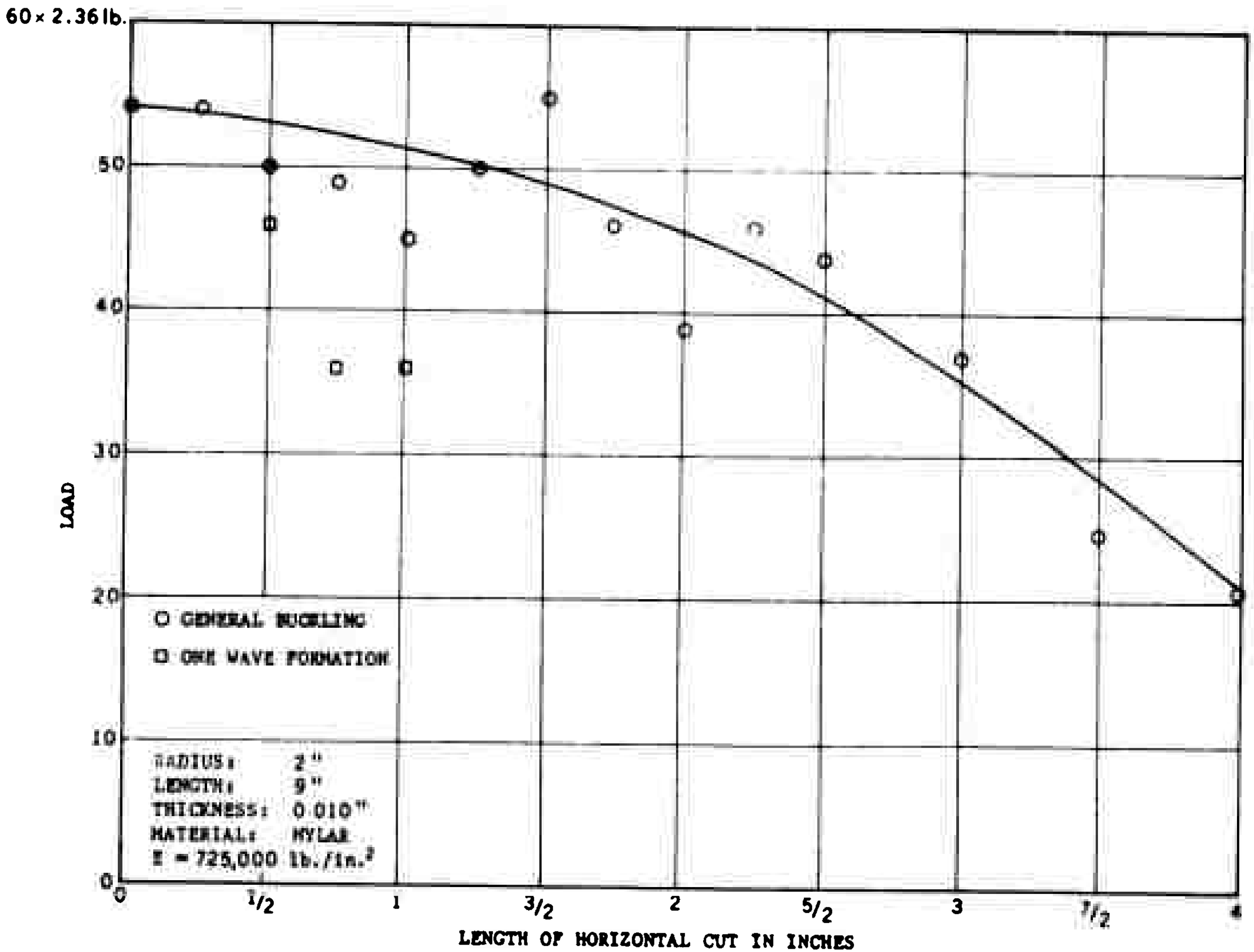

FIGURE 7 REDUCTION IN BUCKLING STRENGTH OF A MYLAR CYLINDER DUE TO A CIRCUMFERENTIAL CUT

VIII SUMMARY AND CONCLUSIONS

A theory of acoustic fatigue is proposed which is based on a general theory of fatigue of metals. The theory is based on the proposition that the problem is better understood if attention is focused on the growth of the crack which is responsible for the ultimate failure. One significant manner in which the present theory differs from others is, in that it invokes a stress dependent limiting condition on the crack growth to determine the cycles for failure. Crack growth expressions are derived on a semi-intuitive basis and these are used for defining equivalent damage at different stress levels. Representing the result of acoustic input as one of random stress response, expressions are derived for the prediction of time to failure. Indications are that the rate at which the damage is accruing in acoustic fatigue is strongly dependent upon the low stress end of the distribution function, and the failure conditions and the associated probabilities of failure are strongly determined by the high stress end that is, the tail of the distribution function. Another important result of the analysis is that in any acoustic fatigue simulation tests, one has to take into account the design mean stress for which the structure is designed. This design means stress not only influences the total time to failure but it also strongly affects the probabilities of failure by essentially changing the origin of the distribution function curve.

IX RECOMMENDATIONS

1) The current practice of describing the stress response of panels in acoustic fatigue in terms of a power spectrum and the associated frequency distribution of the r.m.s. stresses is open to question. Such a description may be adequate for some aspects of response of structures to random excitation, but it does not appear to be adequate in the treatment of the problem of acoustic fatigue damage. Physical considerations of damage indicate that both stress range and peak amplitudes play an important part in determining damage and complete failure. It is therefore recommended that serious consideration be given for a mathematically suitable and physically adequate description of the stress response in acoustically induced vibrations from the standpoint of fatigue damage.

2) The probabilities associated with the high stress amplitudes play a fundamental role in determining when complete failure would occur. These are normally described by the tail of the distribution function. Because of their importance in determining failure, a careful study of the tail of the distribution function associated with the r.m.s. stresses is of paramount importance.

3) The current practice of describing response in acoustic fatigue in terms of an r.m.s. stress cycles to failure is highly unsatisfactory for at least two reasons. In the first instance, the stress input is essentially random and therefore the damage response must necessarily be probabilistic in nature. This aspect does not explicitly appear in the current practice. Secondly, in acoustic fatigue, the panel is, so to say, simultaneously subjected to damage at different frequencies. Therefore, it is recommended that time to failure be considered a more appropriate independent variable for acoustic fatigue and instead of giving an r.m.s. stress vs. cycles curve, curves of r.m.s. stress versus time to failure for various probabilities be given to describe acoustic fatigue damage. In this report, a semi-theoretical method has been presented for such a description. Consideration may be given to this and other similar methods of approach.

4) An implicit item of knowledge in the treatment of cumulative damage in fatigue is a basic σ vs N curve. A knowledge of this is presumed for prediction of damage due to intermixing of stresses. It is recommended that the fatigue response of a typical panel excited in its fundamental mode at various stress levels be considered as the basic information for predicting acoustic fatigue damage. An experimental setup where the fundamental mode is excited fairly simply is highly desirable for preliminary tests, before expensive tests are initiated in the new acoustic test facility of the RTD. Such an experimental setup must also have means for applying mean stresses in the panel. Means of producing biaxial mean stresses in the panel are highly desirable.

5) It is recommended that any tests in the new RTD acoustic test facility intended for establishing design criteria and standards must always try to simulate the anticipated design mean stresses, if the data is to be of any use.

6) Acoustic fatigue induced reduction in stiffness and hence in buckling strength, much like creep buckling, is a possibility. Present indications are that reduction in strength is more critical than reductions in stiffness. However, tests on simple cylindrical shells subjected to compressive loads and acoustic radiation are needed to explore this problem further.

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